THE 41st—13th INTERNATIONAL—RUDOLF ORTVAY PROBLEM SOLVING CONTEST IN PHYSICS
22 October—2 November 2010

1. Followers of the Old Empire just don't know when to give up. After years of planning and several failed prototypes, the 42nd Death Star is operational. The Death Star leaves a secret base and travels with constant velocity. Luckily the Republic intelligence discovers this and they decide to destroy both object. They even manage to obtain a bit of information on the Death Star itself. The data obtained is used to operate a very elaborate instrument that does the following: for a manually set point in space, the machine determines whether the Death Star is within a range of one parsec of it. Rather unfortunately the machine consumes a lot of power and it takes one second to charge it up for the next 'shot'. If the Republic can't get close enough to the base before turning on their sensors, the secret base might notice their presence and change its location. To prevent this, the base has to be located with a precision of $R \ll 1$ parsec, before the fleet is deployed. How many dimensions can the space have for this to be possible? How do they find the base? On reaching the base, it is possible that the followers of the Old Empire contact the Death Star advising it's crew to change their path. If the Republic could locate the Death Star with sufficient precision (R) before invading the base, they could simultaneously attack both. Can they locate the Death Star, or do they have to rely on luck and hope that no transmission is sent to notify the Death Star of their presence? If they can't locate both simultaneously but the Force is with them and no transmission is sent, can they locate the Death Star with sufficient precision after the invasion of the base?

   (Máté Balogh, based on the idea of Lajos Pósa)

2. Iceball is a perfectly spherical asteroid, which rapidly spins around its axis. Its surface is smooth ice, free from bumps or cracks. Underneath the ice, valuable minerals and signs of foreign civilizations are suspected to hide. It is well understood, than human beings wish to search through every square inch of the surface of the planet, therefore they build two bases on the surface.

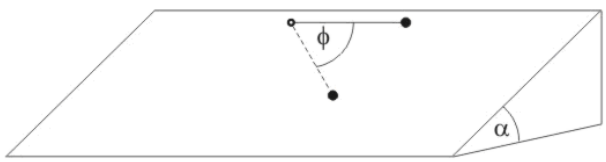
   The research stations are deployed – after a number of failed trials – exactly at the poles of the asteroid (otherwise they would slip away on the ice due to the centrifugal force). They figured out a very simple method to travel between the two polar stations: the cargo is attached to ice skates, the station's door is simply opened, and the object is pushed in the proper direction with proper speed – and let it slip!

   a) Which is the orbit of the cargo (in a coordinate system fixed to the rotating planet)?
   b) How should one choose the initial velocity, if the cargo is designed to pass right through the open door of the other polar station (the door is on the same meridian on both stations)?
   c) Ice Joe, the technical coordinator of the station introduces an excellent innovation. He suggests to spin the cargo at the instant of the push at angular speed of ω around the vertical axis. His statement is that with this operation it is achievable that the cargo returns to the base camp after having wandered all around the planet discovering reasonable fraction of its surface. (Evidently, this renders the maintenance of the other station unnecessary, and the money that is spared can be spent on new Earthian video tapes). Let us try to make a calculation, if Joe is right! How many tapes can Joe buy from the saved money?

   Concerning technical data, the radius of the planet is R, it spins at an angular velocity Ω, the surface gravitational acceleration is g, the diameter of the polar stations is 4 seconds of a degree (in the geographic – or icegraphic – grid).

   (Gyula Dávid)

3. One end of a thin thread is fixed to a ramp with declination angle of α, and to the other end a point-like object is attached according to the Figure. The body is released with zero initial velocity from a position at which the thread is straight and horizontal. The frictional coefficient between the ramp and the body is μ. Determine in an 'elementary' way, that is, without differential equations and only using the proper physics analogy, at which position of the thread will the force along the thread be maximal!



   (László Pálfalvi)

4. A skier is sliding downhill on a slope with inclination α. If he reaches a small bump, and passes through with no change of body position, he can continue sliding after a (small) jump. It is well known among professionals, that they are faster if they do not, or if not possible, they jump as small as possible; that is, sliding on the snow is faster than jumping. Which is the physical basis of this surprising phenomenon, supported by experience? Let us neglect the fact that during the jump the skier needs to steer himself, which may result in increased air drag, but take into account that the frictional coefficient is finite (e.g. $\mu = 0.05$).

   (Gábor Cynolter)

5. Investigate the velocity-dependence of the drag force using a pendulum. Suspend a heavy ball (sphere) on a long string, and let it swing freely in the air (we will neglect the drag force on the string). Let us measure the total mechanical energy of the pendulum (deduced from the position of the turning points) as a function of the number of periods (swings) completed since it was started. This provide the energy loss suffered in one single period (swing). Let us calculate this energy loss per period assuming various simple functional forms for the velocity dependence of the drag force (e.g., linear, quadratic, etc.), and compare the calculations to the measured data. Use analytic calculations where possible. Try to plot the measured data in a simple and intuitive form, and demonstrate the comparison between measurements and calculations graphically. To explore the velocity interval better, perform the measurements at various pendulum lengths.

(Gábor Veres)

6. Since around here we do not have mirages during wintertime, Balint and Rose had to find something else to spend their afternoon with. In the end they settled for their recently acquired Matchbox cars. Balint is wise and cautious, he takes good care of his cars and only gives them a small initial velocity. Rose on the other hand is obsessed with speed, so she tries to push the cars as fast as she can. Their track, which lies in a vertical plane is designed so that the cars remain on the track from start to end. They do not mind the friction but none of them like air resistance, they promptly put together a vacuum-chamber for the track. They both are very keen physicists so they quickly derive the relevant formulae for the terminal velocities of the cars as a function of the initial velocities, based on the shape of the track. What did they get? They are so pleased with their results, they cannot resist telling Violet about them. Violet does not know much about differential equations. Still she tells them the results aren't surprising at all and what is more, she could have derived almost perfect expressions for the terminal velocities in her head. What did Violet refer to? Balint and Rose were so taken aback with Violet's reasoning that their faith in this being the greatest thing since the 42-card Tarock, wavered slightly. In order to calm down, they need to see the explicit solution for an interesting track to verify they get their results in the relevant limits. Help them with this! Is it possible to observe both limits on a track using real Matchboxes?

(Máté Balogh)

7. If you ride too slow, you fall over – as it is well known, if one rides the bicycle too slow, it is easy to capsize. Let us model this (in reality rather complicated) phenomenon by a single hoop of mass m and radius R (the mass of the hoop is concentrated on the very rim), which rolls vertically on a horizontal ground at velocity v_0 without slipping. Which is the minimal value of v_0, if this motion is stable against small perturbations? Which is the frequency of the vibrations (waggling) of the hoop, if it is slightly pushed out of its original motion?

(Péter Gnädig)

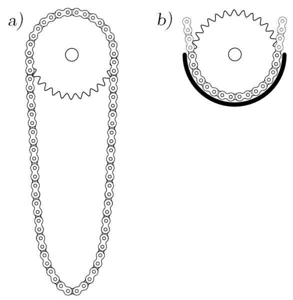
8. For suspension bridges, both the suspending cables and the bridge body represent sizeable weight. Which is the shape of the cable in this case? Study the special cases, when a) the bridge body is much heavier than the cable and b) vice versa. The two cases correspond to the 'short' and 'long' bridges. For a given spanned length of the bridge, in which case does one need longer cable? Study the Erzsébet-bridge in Budapest as a realistic example!

(József Cserti and Géza Tichy)

9. A rigid body is equipped with two fixing points. To the fixing points, we attach the two ends of an ideal flexible and smooth rope of given length, and we hang the rope on a hook. The rope can slip on the hook without friction. Which conditions must be fulfilled so that the suspended body is in a stable position? Is it possible to hang a homogeneous sphere in the way discussed above, with attachment points on its surface, so that the ropes are touching the sphere in the attachment points only, and its position becomes unstable?

(Zoltán György Radnóczi, based on the idea of Gábor Stépán)

10. The axis of a gear is fixed horizontally, a bicycle chain is placed over that according to Figure *a*, and it is started to rotate, spinning up slowly.
a) Which is the shape of the chain, when the gear spins fast with a constant angular velocity?
b) Using appropriate structure to guide the shape of the chain, one can achieve a situation when the chain touches only the lower side of the gear (and so the center of mass of the chain is initially above the gear) as shown in Figure *b*. In this case, is there a possible initial shape of the chain, in which the shape of the chain is stationary? In other words, find the theoretically possible (stable or unstable) steady state positions of the chain.



(Máté Vigh)

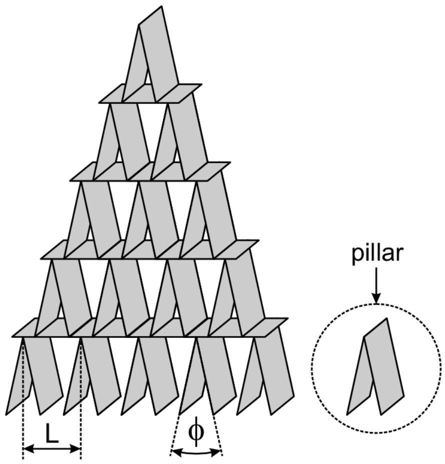
11. Is it possible that a body which falls (tilts) without push from its original, unstable equilibrium state on a horizontal, frictionless table, detaches from the table surface during the motion?

(Máté Vigh)

12. A perfect card tower is being built from perfect pieces of cards. The pieces are all equivalent, with straight edges, negligible thickness and homogeneous mass distribution. For the geometrical and physical parameters (size, mass, frictional coefficient) one can take reasonable estimates.

The card tower is built on a ground of the same material as the cards. In the tower, in each pillars of an upside-down 'V' shape the pair of cards take the same angle ϕ between them, that is, each stages are equally high. The pillars are placed at equal distances at each stages, and the adjacent pillars may touch each other.

On the top of each pillar pairs, there is a cover piece, which is properly placed in the middle position. Let us assume that the cards do not bend. Determine how high can such a card tower be, if the local geometrical structure of the tower is fixed: that is, the distance of the pillars and the angle ϕ is constant. How many packages of cards does one need for the highest tower?



Evidently, reducing the angle ϕ, an arbitrary high tower could be built in principle (in the limiting case, from pillars of vertical cards), but such a tower will be more and more unstable. Assume that the card tower is built from stage to stage, from the left towards the right. We are very precise during the work, no shaking hands, and so in homogeneous gravitational field we can go on with building for an arbitrarily long time our perfectly vertical tower. It might happen however, that the last pillar we place, falls. Let us model this falling by assuming that it starts at zero initial velocity, and the cards will collide inelastically with the adjacent pillar or with the cards forming the base of the stage. We would like to build the card tower in such a way, that in the case when the last pillar falls before the placement of the horizontal cover piece, it would not make any other covered pillars fall in consequence. Assume that the placement of the final cover piece is safely manageable. With which parameters (pillar distance and angle ϕ) can one make the highest tower? If a numerical calculation is performed, give the solution for at least two different values of the frictional coefficient.

Can we improve the theoretical achievable height considerably, if we do not insist on the equality of the angle ϕ and the equal distances of the pillars on each stages, and also if we can place the cover pieces arbitrarily on the pillars? Is it important in what sequence do we go on with the building of the tower? Let us study if the assumption that we neglect the bending of the cards is acceptable. Compare the results of the studies with the actual world record.

(Előd Merse Gáspár)

13. Let us try to estimate the tensions which could possibly lead to the collapse of the red sludge reservoir (Ajka, Hungary, 4 October 2010). Why did the dam breach through just at the corner? Does the fact, that the dam slipped at many places, suggest anything? Consider the dam as a ribbon with high elastic coefficient, and estimate which tensions will emerge in a reservoir of arbitrary shape, if one assumes the strength of the attachment to the carrying soil to be a free parameter. In which case is it a bad idea to build a triangular or square shaped reservoir?

(Péter Hantz)

14. Consider an empty, sealed PET bottle. As the temperature is decreasing, the pressure of gas decreases accordingly and the bottle will be deformed. Depending on the difference of pressure between gas in bottle and the atmosphere the transverse cross-section of bottle takes a polygonal shape. At high pressure difference it will be a triangle (at extreme high pressure difference it becomes fully oblate, a digon), and as the pressure of gas in bottle is increasing, the polygonal shape transforms into a tetragon, a pentagon and in the limit of zero pressure difference it will be a circle. The phenomenon obviously depends on the material of the bottle and thickness of its wall. Let the PET bottle be a perfect one, i.e. its material is homogeneous, isotropic and elastic; the thickness of its wall, r is constant, its shape is cylindrical, the radius and height values are R and h, respectively, $r \ll R$ and $h = kR$, where $k \approx 6$. Give the expression of energy levels E_n, where n is the number of sides of a polygon assuming that the gas in the bottle is ideal. Give the value of E_n in the case of $n \to \infty$. What can be said in this case, about the density of energy levels?

(Gábor Homa, Krisztián Kis-Szabó and Gergely Dobrik)

15. Is it possible that two toroidal, rigid, homogeneous planets are forming a system in which the toroids are chained through each other, and during their mutual motion, the planets are never touching each other? Study the systems of the above kind, and classify them according to their geometrical parameters, masses and initial dynamical parameters. Are there systems which are stable? (Consider the fact that there are a sizeable number of dynamical degrees of freedom.) What happens in the case for example, if the 'orbit' frequency reduces a bit, or if the plane of one of the toroids tilts slightly? (Analytic calculations are favorable. For numerical calculations, the results should be cross-checked in some semi-analytic way.)

(Előd Merse Gáspár)

16. Let us try to model a copper church bell by a spherical shell, with inner and outer radius of R_1 and R_2. Study the elastic waves which appear in the bell. Give an approximation for the smallest frequency emitted by a bell of typical size!

(József Cserti)

17. The elastic properties of homogeneous, non-isotropic crystals can be expressed by the elastic coefficient tensor C_{klpq}. In such crystals in general, for any wavenumber vector $\mathbf{k}$ there exists three mutually perpendicular polarization vectors $\mathbf{a}^{\beta}(\mathbf{k})(\beta = 1, 2, 3)$, neither of which are parallel or perpendicular relative to $\mathbf{k}$. These polarizations determine the displacement vectors of the plane wave modes, to which different vibration frequencies, and hence different phase and group velocities are belonging.

Let us start a wave packet constructed around $\mathbf{k}_0$, concentrated on one of the polarizations ($\beta = 1$). Study the phenomenon from a frame of reference which travels at the speed of the wave packet (that is, the group velocity corresponding to the vector $\mathbf{k}_0$). Imagine a situation when our detector is only sensitive to an other polarization, (say $\beta = 2$). What can such a detector measure? What is the position and time dependence of the signal detected in the $\beta = 2$ mode?

Extra question: Which interesting quantum mechanical effect does this phenomenon resembles?

(Gyula Dávid)

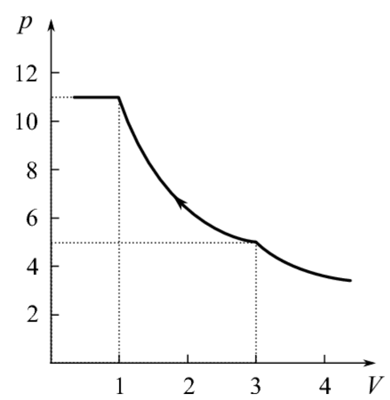
18. In medium sized waterfalls it is a common phenomenon, that on the lower water surface not only bubbles appear, but also drifting water droplets. These water balls are moving around on the water surface, then they merge with the water mass.

Let us construct a theoretical model, which can determine the movement and lifetime of the balls. To what extent does this phenomenon depend on the composition of the liquid? What happens with the lifetime, if the water surface moves periodically? How do the droplets interact between each other on the water surface?

The phenomenon can be well realized with dripping soapy water, and therefore the calculations may be supported by measurements.

(István Szécsényi)

19. In a closed container, equipped with a piston on one end, a mixture of oxygen and nitrogen gas is stored at temperature $T = 77.4$ K. The total mass of the gas is 176 g. (In fact, at normal atmospheric pressure, the boiling point of nitrogen is $T = 77.4$ K, whereas the boiling point of oxygen is higher, 90.2 K). At constant temperature, we slowly compress the gas mixture, while the pressure increases as a function of the volume according to the Figure, in relative units.



a) What is the mass of the oxygen and nitrogen component in the container separately?
b) Based on the given information, estimate the latent heat of vaporizing oxygen, if the vaporization heat of nitrogen is known to be 199.3 J/g.
c) After finishing the compression we increase the temperature at constant volume to $T_2 = 90.2$ K, then we let the gas mixture slowly expand isothermically. Plot roughly on the p–V plane the isothermal line which describes the expansion, and mark the values of pressure and volume in SI units for the distinctive points.

(Gyula Honyek)

20. The combined cycle heat engine is constructed from two heat engines, connected in series. The first, usually a gas turbine, operates between a high temperature heat storage and a lower temperature point. The heat delivered here by the first engine drives the second heat engine (generally a steam turbine), with the external environment acting as low temperature heat sink. Assume that the efficiencies of both of the two engines, as well as the unified heat engine, is the same function of the two heat storage temperatures. Show that in this case, the engines realize the ideal Carnot cycle.

(Géza Tichy)

21. The eccentricity of the Earth's orbit changes between $\varepsilon_{min} = 0.005$ and $\varepsilon_{max} = 0.058$ on a timescale of 100 to 400 thousand years. Due to the varying eccentricity, the incoming radiation from the Sun changes and, in principle, this may result in climatic changes on Earth. Let's calculate the maximal change in the incoming radiation due to changing eccentricity, and let's estimate the resulting change of the temperature on Earth assuming that the Earth reradiates the incoming radiation as a black body.

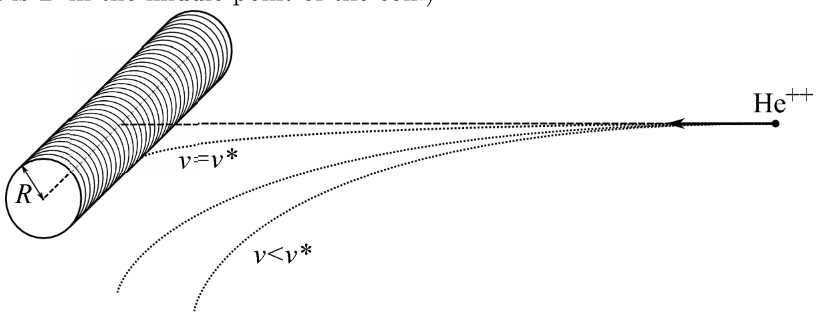
(Zoltán Rácz)

22. It is a well known fact, that if electric current flows along an infinitely long straight conductor in vacuum, the magnetic field lines are circles. Is it possible that a circular field line appears in such a magnetic field, which is generated by two infinitely long straight and parallel conductors, which are at a given distance from each other in vacuum?

(Gyula Radnai)

23. In a cloud chamber the motion of α particles is studied. The beam of particles is emitted towards a solenoidal coil, which is of radius R, length L, in the symmetry plane perpendicular to the coil axis, in the direction of the middle point of the coil, according to the Figure. The particles are emitted from a relatively large distance, say from $10L$.

One finds that due to the external, stray magnetic field of the coil the beam will reach the coil only if the velocity of the particles is higher than a critical value v^*. What is this value? (Study the problem in a classical fashion, and assume that the particles move always in the symmetry plane perpendicular to the coil axis. The value of the magnetic field is B in the middle point of the coil.)



(Máté Vigh)

24. As part of the Christmas preparations, Edford is painting a glass orb with golden paint. He's not really creative, so he is thinking about simple geometric patterns. In the end he decides to paint a curve on the sphere, which intersects all the meridians of longitude at the same angle. He paints the curve all the way from the 'South pole' to the 'North pole'. Since Edford is from a rather wealthy family, the golden paint he uses is actually made of gold. (We can neglect certain unimportant details such as the curve coming close to overlapping itself.)

a) Calculate the current passing through the pattern when Edford links the 'poles' to a 9V battery. Calculate the magnetic field B in the centre of the sphere.

b) Edford's sister, Edweena is a little clumsy, so when she takes a closer look at the decorative orb, she drops it to the ground. Upon hitting the floor, the orb splits into two along the perpendicular bisector (plane) of the line segment connecting the two poles. Edford is disheartened by this event, however he cheers up once he notices how pretty the shadow is cast by one half of the orb (it is illuminated by a point light source far away along the line originally joining the poles, and the shadow is cast upon a wall parallel to the plane of the crack). He promptly paints the shadow with his golden paint. Give an estimate as accurate as you can for the amount of paint necessary for this.

c) One end of the curve is the image of the pole. A current I is passed through the other end of the curve and a point close to the image of the pole. Give a good estimate of B at the image of the pole. How good are your estimates in case of a realistic Christmas decoration?

Bonus question: An old friend of the family, who is a retired Shipmaster comes to visit during Christmas season. He falls into a happy state of nostalgia upon seeing the orb painted by Edford. How old is the captain?

(Máté Balogh)

25. Last week we have been visiting planet 'Alexander'. One afternoon during one of our long excursions (enjoying the atmosphere, which is same as on Earth, but much clearer) we have observed a wonderful rainbow. Unlike Earthian rainbows, this had both the inner and outer arcs of violet color, followed by the other colors of the visible spectrum, and in the middle there was a broad red band. What could possibly be the refractive index of the (spherical) droplets floating in the cloud? Which was the observation angle of the middle band of the rainbow?

(József Cserti and Gyula Dávid)

26. Calculate and display the shape of the caustics outside, and as well inside of a water droplet, arising when the principal rainbow formed. Assume that the droplets are spherical.

(József Cserti)

27. In the famous letter written by Einstein to Roosevelt in August 1939 there is a sentence which reads: 'However, such bombs might very well prove to be too heavy for transportation by air.' Why could Leo Szilard, Eugene Wigner and Edward Teller, who all contributed to the formulation of the letter, consider such issue?

(Gyula Radnai)

28. In amorphous materials the Voronoi polyhedron can be defined for each atom, with its inner points being those which are closer to the atom in question than to any other surrounding atom. Show, that for the Voronoi polyhedrons, the following condition is fulfilled:

$$\sum_i (6-i)n_i = 12,$$

where n_i is the number of sides with i edges!

(Géza Tichy)

29. In metals, the heat conductivity of the lattice is much smaller than that of the electrons, and hence the Wiedemann–Franz law holds. However, this is also true for heavily doped semiconductors, where the Wiedemann–Franz law fails. Which basic conditions must hold in order the Wiedemann–Franz law to be fulfilled?

(Géza Tichy)

30. Determine all the possible paths which a light beam can trace in the gravitational field of a spherically symmetric body. Does any closed path exist?

(Gyula Bene)

31. Four equivalent spaceships are falling towards a non-rotating black hole of diameter of 2000 billion km (US billion, i.e. $2 \times 10^{15}\, m$, this is the size of the event horizon), all in the same plane, along lines which are perpendicular to each other. The velocities of the spaceships were zero at infinite distance. They all cross at the same moment the spheres of any given radius drawn around the black hole ('the same moment' is calculated in the coordinate system according to the Schwarzschield metric). The spaceships are of same size with 1 km length, and a radio transmitter at the bow of the ships emits a sharp radio signal in each microsecond.

During the fall – having nothing better to do – the crews of the space-ships are spending their (proper) time with observations of the other ships. Calculate, in which directions do the passengers of any ship see the others. (The phrase 'when' is defined by the proper time of the spacemen, and the zero time is fixed to the crossing of the event horizon).

Study the time period before and after crossing the event horizon. How and when do the astronauts observe the spaceship approaching them from the front? On which orbit do they see the neighboring ones moving? What is the observed length of the other ships as a function of their proper time? How do the period of the radio signals emitted by the other ships vary, again as a function of their proper time? Can they exchange messages or radio signals before and after crossing the event horizon?

(Gyula Dávid)

32. The Dirac Hamiltonian describing massless spin-half-particles moving in the $\mathbf{r} = (x, y)$ plane is given by

$$\hat{H} = v\,\boldsymbol{\sigma} \cdot \mathbf{p} + V(\mathbf{r})\,\sigma_z,$$

where $\boldsymbol{\sigma} = (\sigma_x, \sigma_y)$ and σ_z are the Pauli matrices; $\hat{H}$ acts on a two-component column spinor wavefunction $\psi(\mathbf{r}) = (\psi_1(\mathbf{r}), \psi_2(\mathbf{r}))^T$, ($T$ is the transpose of a vector) and v is a constant. The mass-related potential $V(\mathbf{r})$ is coupled to the Hamiltonian via the Pauli matrix σ_z. Consider a ring shape domain D ('billiard') with inner radius R_1 and outer radius R_2. Assume that the potential $V(\mathbf{r}) = 0$ if $\mathbf{r} \in D$ (inside the billiard) and it tends to infinity at the boundary of the domain D (this is called hard wall bounding). In this way, particles are confined inside a ring shape billiard and it was shown that the boundary conditions are $\psi_2/\psi_1 = \pm i e^{i\varphi}$, where φ is the polar angle of the vector $\mathbf{r} = R_2(\cos\varphi, \sin\varphi)$, while $+$ and $-$ correspond to the outer and inner circle of the ring (see, e.g., M. V. Berry and R. J. Mondragon, Proc. R. Soc. London, Ser. A **412**, 53–74 (1987)). Derive the secular equation determining the positive energy eigenvalues of the above Dirac Hamiltonian. Calculate the first few energy levels (in units of $\hbar v/R_1$) numerically for a given ratio R_2/R_1. What is the physical background of the boundary conditions given above? Is there any relation of this problem to the neutrino or the well-known graphene?

(József Cserti)

33. Analysis of the absorption spectrum of a single particle in three-dimensional quantum boxes with infinite high barrier energy. Give a brief analysis on the effect of absorption spectrum of a single particle in three dimensional quantum boxes with the *same* volume but different shapes if the particle was initially in its ground state. Consider a cube with edge a and a general rectangular box with edges b, c, and d. What is the absorption spectrum of the cube and a general rectangular box? (Use the Fermi golden rule in the calculation of absorption spectrum.) Take a special case of a rod-like rectangular quantum box ($d = \chi a$, $b = c = a/\sqrt{\chi}$). Describe the polarization dependence of the absorption spectrum in terms of the resonance frequencies and intensities compared to the special case of cube. Which shape of quantum box has overall larger absorption in the range of excitation energy between zero and 100 times of the ground state energy of the quantum cube when χ is a *bit* greater than 1?

(Ádám Gali)

34. A sharp needle is placed over a grounded semiconductor sample, which can be considered as a p–n reverse biased junction (diode with voltage applied in the non-conducting direction). If a sufficiently high voltage is applied on the needle, there will be an ion channel forming due to the electric field, which transfers charges on the surface of the semiconductor. This results in a time dependent potential on the surface of the sample. Above a specific threshold voltage, these charges will leak towards the ground from the surface and removed; such effect is not classical.

Let us assume that we apply a periodic square-wave voltage on the needle, that is, we charge it for time t_1, then for period of t_2 we determine the surface potential with a sufficient sensor, and this is repeated n times. Once we cross over the threshold voltage necessary for initiating the leakage current, the surface potential will be described as a function of time in the following way: At the beginning of the charge the potential is U_1, at the end of the charge it is U_2, at the end of the measurement it is U_3. Let U_k denote the threshold voltage which initiates the process.

During the charge up, the following equation holds: $C\frac{dU}{dt} = -I_0(e^{\frac{U-U_k}{V}} - 1) + I_f$, where C is the capacity of the sample, U is the surface voltage of the sample as a function of time, V is the thermal potential at temperature 300 K, I_0 is the current at the beginning of the process, I_f is the source current through the needle. When the sample is not charged, $I_f = 0$ must be substituted in the equation above.

Solve these equations, and plot the first $n = 10$ periods with freely chosen reasonable parameters.

(Gábor Homa)

35. While mummy is breast feeding, daddy goes on calculating...
Breast feeding is a wonderful, but evidently not an easy task for any mother. Can possibly help in this instinctive but at the same time learnable process – besides the specific literature, the nurse, the breast feeding advisory, the doctor of infants, the all-knowing relatives, the highly trained young mothers owning two-months-old children, and the internet standing far above all – the physics or mathematics? We are looking for the optimal feeding strategy from two breasts, in a seriously simplified model with the following assumptions: i) Both breasts can store the same amount of milk. ii) The flow rate of sucked milk is proportional to the amount stored in the specific tit. iii) The milk production rate is proportional to the amount of missing milk in the tit.
a) How do the amount of actually stored and the consumed milk vary during the breast feeding process?
b) What is the optimal, one-time feeding strategy from two breasts, during a time interval of T?
c) What is the optimal continuous feeding strategy, if between each feeding time interval T_1 there is a rest time of T_2?
As the baby's body is developing, the feeding becomes more and more effective, and if everything goes well the amount of milk production increases accordingly. How do the optimal feeding strategies depend on the ratio of the sucking speed and the milk production speed?
Additional useful information about breast feeding can be found at http://www.llli.org/ :)

(Gábor Vásárhelyi)

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